Financial Modeling and the Crisis II: Asset Allocation

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1. Introduction

Following the 2007 – 2009 financial crisis, a sharp and sometimes acrimonious debate has ensued about the usefulness of financial models in the throes of that crisis, including their value in making asset allocation decisions. Few will dispute the basic stylized facts that describe the crisis: as asset prices dropped sharply, the volatility of asset returns increased substantially; correlations between returns both within and across asset classes increased substantially as market factors increased in relative importance;[2] and, as recent survey evidence suggests, average risk tolerance decreased as wealth decreased and risk concerns became more salient. As the crisis unfolded, a big question for most investors was what tactical adjustments they should make to their portfolio holdings given the large losses in wealth they suffered and the extreme conditions they faced in the markets. In this article we address this question by considering how the simple economics of supply and demand play out in a situation where both risk and risk aversion have significantly increased. We do this by developing a very simple model calibrated to capture the stylized facts mentioned above, and we consider within that model how investors should trade among themselves as conditions change.

Perhaps the most natural response to a crisis, at least for many investors, is to "flee to quality" as confidence in the market erodes and prospects dim, especially for equities. Of course, any investor who flees to safety must convince another investor to take the other side of his trade and "flee" to increased risk. This means that only a subset of investors can flee to safety. Our analysis is based on this simple observation, one that is unfortunately quite often forgotten (or ignored) in discussions of asset allocation and tactical responses to changed market conditions. In a crisis asset prices must adjust so that a substantial number of investors find it in their interest

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[2]For example, common factor risk as a fraction of the volatility of the typical U.S. stock's return roughly doubled from 30% as late as April 2007 to 60% in early November 2008: Source: Quantal's conditional variance-covariance matrix for U.S. equities at the beginning of these respective months.

to hold risky assets despite the increased uncertainty in the economy. Indeed, market equilibrium essentially requires that the "average" investor be willing to hold the available assets, including risky assets, in roughly their market proportions. This does not mean that no investor will want to make adjustments when market conditions change. Many will. But for anyone wanting to make a change in one direction, there must be someone else willing to make the change in the other direction. As shown below the direction and size of the optimal adjustment for any particular investor depends on individual circumstances. One of the most important is the investor's tolerance for risk relative to that of the average investor.[3] In the simple example we discuss below, investors who have a lower tolerance for risk than the average investor, i.e. a higher ratio of fear to greed, want to adjust their portfolios to be more conservative when risk increases, while those who are more risk tolerant than average want to make adjustments that expose them to even more risk but allow them to earn commensurately higher prospective returns. In other words, the fall in risk asset prices in the crisis is sufficient to create a tradeoff between risk and return that motivates the more risk tolerant investors in the economy to actively take on more risk, accommodating those investors who want to reduce their risk exposures.

As mentioned above, our example depicts a crisis similar to the one we recently experienced. We assume that once the crisis occurs, both asset volatilities and correlations increase substantially. These increases capture the higher uncertainty and the increased importance of common risk factors in a crisis environment. We also assume that risk tolerances decline across investors. A key consideration for how an investor should respond to these changes is the relation of that investor's risk tolerance to the average investor's risk tolerance. This average should be thought of in terms of a wealth-weighted average, not a simple average. Because the average is wealth weighted, a financial crisis such as the last one leads to a change in the societal average risk tolerance even if no individual investor becomes less risk tolerant. This is because more risk-tolerant investors generally will have more risky holdings going into a crisis, and because of this will suffer greater decreases in their wealth relative to more conservative investors. The distribution of wealth will shift toward the less risk tolerant. Due to this rearrangement of wealth, there is an endogenous drop in average investor risk tolerance beyond that due to any changes in individuals' risk appetites. The opposite, of course, tends to occur in bull markets when risky assets appreciate in value.

In our calibrated example we find that risk premiums on both equities and bonds approximately triple in response to the crisis conditions we assume. Increases of this magnitude are required in order to clear the market. Despite these large changes in risk premiums, our example shows that the magnitude of the rebalancing appropriate for most investors is not large, contrary to what many might believe. Average turnover rates are about 7% for the base case we consider.

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[3] Risk tolerance can be considered a measure of an investor's "risk appetite,""risk sentiment" or "fear index."

This finding that tactical adjustments are relatively small, even in a major dislocation such as the last crisis, is quite robust across a number of assumptions we make about what happens in a crisis and the opportunities available to investors. If, for example, we impose restrictions against short selling and leverage, we find, not surprisingly, that the average turnover is even less. We also find that substantially increasing the heterogeneity of risk preferences in the economy has at most a modest effect on turnover. A crisis scenario more severe than that assumed in our base case can by obtained by assuming that the crisis caused more extreme increases in correlations among asset classes (holding everything else constant), since greater increases in correlations reduce the benefits of diversification even further. We find that while increases in correlations have substantial effects on equilibrium risk premiums, they have quite limited effects on turnover and the size of tactical adjustments. For another check on the robustness of our findings, we also consider what happens when a subset of investors follow a naïve "target weight" rebalancing policy, which involves purchasing those assets that have fallen the most in price and selling those that have fallen less than average or have appreciated. Investors with a target weight policy are mechanically following a contrarian strategy and because of this they increase the demand for risky assets in a crisis relative to our base case. This causes the increases in equilibrium risk premiums to be lower than they would be otherwise. However, even if we assume that a substantial fraction of the population of investors mechanically follows such a contrarian strategy, we still find that overall turnover due to tactical rebalancing is low.

Asset prices are determined in part by *ex ante* risk premiums, which can be expected to increase in a crisis due to the factors we have described above. Our example shows that these risk premiums are sensitive both to changes in the risk structure of returns and to changes in risk tolerance. These changes in risk premiums, along with the likely macro drivers of corporate profitability in the crisis, tended to increase the relative importance of market-wide fundamentals in asset price variation in the last crisis. Note that, as a general point, the fall in asset prices that occurs in a crisis will decrease historically measured risk premiums if these historical averages include returns realized in the crisis. Thus historically measured risk premiums go down at a time when we know that forward looking (ex ante) risk premiums are much increased. This means that the rule that it is next-to-useless to use a short history of past returns to estimate future prospective returns is nowhere more pertinent than in a crisis.

In section 2 we present our "base case" scenario. We explain how risk premiums are determined in changing conditions and examine the trading that occurs as investors make tactical responses. In section 3 we consider a number of variations on our base case and show that these variations do not qualitatively change the general conclusions one draws from the base case. In section 4 we summarize the results and give some concluding remarks.

2. Asset Allocation in the Financial Crisis: Base Case

To keep our analysis simple we consider an example with only five asset classes: US Equities, Developed Market Equities (non-US), Emerging Market Equities, Bonds, and Cash, where "Cash" is meant to include instruments like short-run Treasuries, bank deposits and other instruments that can be regarded as risk-free. Table 1 shows the assumptions we make about the market weights and the perceived distribution of the returns of these four asset classes before the crisis.

Table 1

			Correlations				
	Market Weights	Standard Deviation	US Equity	Dev Equity	Em Equity	Bonds	Equilibrium Exp Return*
US Equity	20.00%	18.00%	1.00	0.65	0.60	0.40	7.11%
Dev Equity	22.00%	20.00%	0.65	1.00	0.60	0.35	7.60%
Em Equity	18.00%	30.00%	0.60	0.60	1.00	0.30	9.84%
Bonds	30.00%	10.00%	0.40	0.35	0.30	1.00	4.71%
Cash	10.00%	0.00%					3.00%**

* Average Risk Tolerance = 0.5
** Borrowing Cost = 3.50%

As can be seen from Table 1, our example is calibrated so that before the crisis hit, riskless cash instruments represented 10% of the market, bonds represented 30% of the total market, and the remaining slice of the market was roughly equally divided between equities in the U.S., developed foreign markets, and emerging markets. Volatilities of the risky asset class returns ranged from a low of 10% for bonds to a high of 30% for emerging market equities. Correlations between asset classes were all positive, but not particularly high. Given that our example is only meant to illustrate in broad terms what happened in the crisis, we are not arguing that any of these parameters precisely captures the pre-crisis situation. We do believe, however, that the parameters in Table 1 represent in a rough way the structure of the market in "normal" times.

As Table 1 shows, we assume the pre-crisis rate on cash was 3%, which we take to have been set by Federal Reserve policies. We also assume that any investor who before the crisis wanted to leverage their portfolios could have borrowed at a 50 basis point spread over Treasuries. The last column of table 3 shows the equilibrium required return levels for the four risky asset classes based on our assumptions. These equilibrium expected returns can be thought of as the required returns that equate supply with demand. We know that prices in the markets must be set so that investors are (in aggregate) willing to hold in their portfolios all the assets

available in the market.[4] If prices of assets in an asset class are too high, then expected returns on those assets will be too low and demand for these assets will be insufficient. Conversely, if prices of a set of assets are too low, perceived expected returns will be too high and investors will want to hold more of those assets than is available, i.e., there will be excess demand. To determine the equilibrium expected return levels we must make some assumptions about investors' risk tolerance, since risk tolerance is related to risk premiums. Risk premiums are lower when investors are more tolerant of risk than they are when investors are less risk tolerant. Since one of our goals is to determine how investors should trade and adjust their portfolios when there is a shock, we explicitly account for the fact that not all investors are the same and in particular differ in their risk tolerance. We assume that before the crisis the average investor had a risk tolerance level of 0.5, but that there was a cross-sectional distribution around this.[5] Our particular assumptions about the distribution of investors by risk tolerance are shown in the first few rows of Table 2.

As shown in Table 2, we assume seven different "clienteles" of investors, where clienteles are distinguished by risk tolerance. The largest clientele is composed of investors with risk tolerance equal to 0.5, the societal average. This group is assumed to control 30% of the total wealth portfolio. Clienteles with risk tolerance levels of 0.4 and 0.6 control respectively 20% of the total wealth. The least risk-tolerant clientele (whose risk tolerance equals 0.2) has only 5% of the wealth, which matches the wealth of the most risk-tolerant clientele (whose risk tolerance equals 0.8). Now assume for the moment that investors on average have the return expectations given in the last column of Table 1, the ones we are calling equilibrium expected returns. This means, for example, that on average investors are forecasting the US equity market will return a little over 7%, while emerging markets will return a bit more than 9.8%. Investors in each clientele will want to adjust their portfolios to obtain the best tradeoff between risk and return given their preferences. This will entail identifying the efficient frontier, i.e., the set of

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[4]The model that we use here for asset allocation decisions is the standard Markowitz mean-variance one, but we believe that the points we make would continue to hold in more sophisticated dynamic allocation models, or in a state-space solution framework such as that developed in Sharpe (2007). Our specific calculations here do implicitly assume a setting in which asset return variances and covariances effectively capture most of what is relevant in assessing risk. The most common justification for focusing on second moments in the (conditional) return distribution is that asset returns are to a close approximation (multivariate) Gaussian. Much of the criticism that financial models received post-crisis has been focused on "fat tailed," non-Gaussian behavior of returns in the crisis. This no doubt will lead some to question this Gaussian assumption. However, as explained in Marsh and Pfleiderer (2011), shifts in the underlying uncertainty, as proxied at the S&P Index level by the VIX, account for a significant amount of the fatness in the tails of *unconditional* distribution of returns on the S&P Index. Thus, so long as the increases in the volatilities and covariances of returns assumed in our example are taken to represent the adjustments made in *conditional* forecasts, our use of a Gaussian assumption for these *conditional* returns is not unreasonable.

[5]The risk tolerance level determines the risk penalty an investor assesses for a particular portfolio. An investor with risk tolerance equal to ρ assesses a risk penalty equal to the variance of the portfolio's return (i.e., the square of the volatility) divided by 2ρ.

allocations that give the highest expected returns for various risk levels, and then choosing among these efficient allocations the one that gives the preferred tradeoff between risk and return. Given our assumptions including the posited expected returns, we see that investors in clientele (d), i.e., the average investors, hold about 61% of their wealth in equities, 39% in bonds, and a trivial amount in cash. The most risk averse have substantial holdings in cash and bonds with little in equities, while the most risk tolerant hold a leveraged position, which involves borrowing an amount equal to about 18% of their wealth and investing 118% of their net worth is risky assets.[6]

Table 2

	Clientele	(a)	(b)	(c)	(d)	(e)	(f)	(g)	
	Risk Tolerance	0.2	0.3	0.4	0.5	0.6	0.7	0.8	
	% of Total Wealth	5.00%	10.00%	20.00%	30.00%	20.00%	10.00%	5.00%	
Optimal Allocations	US Equity	8.36%	12.53%	16.71%	20.89%	23.26%	25.88%	29.57%	
	Dev Equity	9.07%	13.61%	18.14%	22.68%	25.84%	29.19%	33.36%	
	Em Equity	7.03%	10.55%	14.07%	17.59%	21.94%	26.18%	29.92%	
	Bonds	15.40%	23.11%	30.81%	38.51%	28.96%	21.72%	24.82%	
	Cash	60.13%	40.20%	20.27%	0.34%	0.00%	-2.97%	-17.69%	
	Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	
									Total
% Holdings in Economy	US Equity	0.42%	1.25%	3.34%	6.27%	4.65%	2.59%	1.48%	20.00%
	Dev Equity	0.45%	1.36%	3.63%	6.80%	5.17%	2.92%	1.67%	22.00%
	Em Equity	0.35%	1.06%	2.81%	5.28%	4.39%	2.62%	1.50%	18.00%
	Bonds	0.77%	2.31%	6.16%	11.55%	5.79%	2.17%	1.24%	30.00%
	Cash	3.01%	4.02%	4.05%	0.10%	0.00%	-0.30%	-0.88%	10.00%
	Total	5.00%	10.00%	20.00%	30.00%	20.00%	10.00%	5.00%	100.00%

Table 2 shows that our posited equilibrium required returns are indeed consistent with supply equaling demand. This is shown in the bottom panel of Table 2, which shows for each clientele that clientele's holdings in the four risky asset classes and in cash as a percentage of total market wealth. For example, consider clientele (c). Investors in this clientele allocate 14.07% of their wealth to emerging markets. Since investors in this clientele represent 20% of total wealth, this means that they end up allocating 2.81% (i.e., 20% times 14.07%) of total wealth to emerging markets. Now if we add up all of the allocations made by each of the clienteles to emerging markets (0.35% + 1.06% + 2.81% + 5.28% + 4.39% + 2.62% + 1.50%), we find a total allocation across all investors of 18% to emerging markets. This is precisely the weight of emerging markets in the total wealth portfolio. In other words, demand for emerging market equity is equal to supply, as it must be. This holds for all four asset classes (and for cash). The posited expected returns on the four asset classes are therefore consistent with pre-crisis equilibrium

[6] Note that this leveraged allocation was for an entirely rational set of expectations, not because the investor was possessed by wildly optimistic animal spirits.

Now consider what happened in 2007-2009: The market value of most risky assets plunged and risk increased dramatically. For purposes of illustration we will assume that equities fell 40% in value across the board and bonds fell 10%, while nominally riskless assets increased in total value by 5%. Again, we are not arguing that this is a precise representation of what happened over a very specific period in the crisis, but rather that it roughly corresponds to the general level of losses and gains in the crisis. As we mentioned above, another likely consequence of the events of 2008 was an overall decrease in risk tolerance.[7] We will capture this by assuming that risk tolerance for each risk clientele decreased by 0.10. The effects of the changes in asset values are shown in Table 3:

Table 3

	Clientele	(a)	(b)	(c)	(d)	(e)	(f)	(g)	
	New Level of Risk Tolerance	0.1	0.2	0.3	0.4	0.5	0.6	0.7	
	New % of Total Wealth	6.24%	11.57%	21.32%	29.27%	18.69%	8.87%	4.05%	
Allocations after 40% Decline in Equity, 10% Decline in Bonds and 5% Gain In Riskless	US Equity	5.47%	8.85%	12.80%	17.48%	20.32%	23.82%	29.83%	
	Dev Equity	5.94%	9.60%	13.89%	18.97%	22.57%	26.87%	33.65%	
	Em Equity	4.60%	7.45%	10.77%	14.71%	19.17%	24.10%	30.18%	
	Bonds	15.12%	24.46%	35.38%	48.34%	37.94%	29.99%	37.56%	
	Cash	68.87%	49.65%	27.16%	0.49%	0.00%	-4.79%	-31.22%	
	Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	
									Total
% Holdings after 40% Decline in Equity, 10% Decline in Bonds and 5% Gain In Riskless	US Equity	0.34%	1.02%	2.73%	5.12%	3.80%	2.11%	1.21%	16.33%
	Dev Equity	0.37%	1.11%	2.96%	5.55%	4.22%	2.38%	1.36%	17.96%
	Em Equity	0.29%	0.86%	2.30%	4.31%	3.58%	2.14%	1.22%	14.69%
	Bonds	0.94%	2.83%	7.55%	14.15%	7.09%	2.66%	1.52%	36.74%
	Cash	4.30%	5.74%	5.79%	0.14%	0.00%	-0.42%	-1.26%	14.29%
	Total	6.24%	11.57%	21.32%	29.27%	18.69%	8.87%	4.05%	100.00%

Since bonds fell by only 10% and equities fell by 40%, bonds have necessarily become a more important part of the total market. They now represent almost 37%, rather than 30%, of the total wealth portfolio. Before the crisis U.S. equities were 20% of the wealth portfolio. After the crisis they are only 16.33%.

[7]For example, Bateman et. al (2010) report that: "Overall, comparing [survey] results between the relatively tranquil asset market conditions of early 2007, and the full-blown financial crisis of late October 2008 suggests a mild moderating of risk tolerance [by Australian individual retirement fund (DC) investors]…." Note that if capital markets are segmented across countries for whatever reason and Australian investors hold predominantly Australian-domiciled stocks, the survey's finding as to risk tolerance shift may have been muted since the Australian financial crisis was milder than depicted in our crisis scenario. We know of no survey evidence for risk sentiment in U.S. or European markets in the crisis.

As we mentioned above, there is a less obvious effect of the change in asset valuations: the less risk tolerant clienteles have become slightly more important (in terms of the wealth they control) and the more risk tolerant clienteles have become less important. Clientele (a) was assumed to control 5% of total wealth before the crisis, but after the crisis controls 6.24%. This increase is the consequence of the fact that this risk adverse clientele allocated less to equities and therefore suffered a lower loss in value than the much more risk tolerant clienteles. This means that the average risk tolerance declines even more than suggested by the 0.10 reduction in risk tolerance that we have assumed for each clientele. Because the clienteles with low risk tolerance have become wealthier relative to the clienteles with higher risk tolerance, the average risk tolerance weighted by wealth levels has declined to 0.385.

Table 4 shows our assumptions concerning the change in the risk structure brought about by the crisis:

Table 4

				Correlations				
		Market Weights	Standard Deviation	US Equity	Dev Equity	Em Equity	Bonds	Equilibrium Exp Return*
Before	US Equity	20.00%	18.00%	1.00	0.65	0.60	0.40	7.11%
	Dev Equity	22.00%	20.00%	0.65	1.00	0.60	0.35	7.60%
	Em Equity	18.00%	30.00%	0.60	0.60	1.00	0.30	9.84%
	Bonds	30.00%	10.00%	0.40	0.35	0.30	1.00	4.71%
	Cash	10.00%	0.00%					3.00%**

* Average Risk Tolerance = 0.5
** Borrowing Cost = 3.50%

				Correlations				
		Market Weights	Standard Deviation	US Equity	Dev Equity	Em Equity	Bonds	Equilibrium Exp Return*
After	US Equity	16.33%	30.00%	1.00	0.75	0.70	0.50	13.53%
	Dev Equity	17.96%	30.00%	0.75	1.00	0.70	0.50	13.63%
	Em Equity	14.69%	40.00%	0.70	0.70	1.00	0.45	17.37%
	Bonds	36.73%	15.00%	0.50	0.50	0.45	1.00	6.40%
	Cash	14.29%	0.00%					1%**

* Average Risk Tolerance = 0.385
** Borrowing Cost = 1.50%

As Table 4 shows, we assume that the volatility of each asset class significantly increased as a consequence of the crisis and the greater uncertainty created by the economic dislocation. In addition we assume that correlations among asset classes rose due to the increased importance of market-wide factors.

What is the appropriate tactical response an investor should make in terms of his portfolio allocation given all of the changes brought about by the crisis? Given that risk has substantially increased and investors' risk tolerance has decreased, it might seem that investors should move to more conservative portfolio allocations and "wait out the storm." Indeed this was the advice that many were given and no doubt there was a "flight to safety" phenomenon. But, as we observed above, not everyone can retreat to safety. Some investors must hold the risky assets. The laws of supply and demand still apply, even in a crisis. Prices of risky assets must decrease and expected returns must increase enough for investors in aggregate to be willing to hold all the assets available. The last column of the second panel of Table 4 shows, for each asset class, the new levels of (annualized) equilibrium expected returns that equate supply with demand. We base these equilibrium required rates on the assumption that short-term riskless rates have been lowered to 1% through central bank (Federal Reserve) policies taken in response to the crisis. If we continue to assume a 50 basis point spread for borrowing, the less tolerant investors will now be able to borrow at a 1.5% rate. We see that equity risk premiums must be approximately eight percentage points higher than pre-crisis, i.e. they approximately triple in size, and the bond risk premium must be more than two percentage points higher, which means that it also approximately triples in size. These increases in risk premiums are due to increased volatilities, decreased ability to reduce risk through diversification (higher correlations), and decreased levels of risk tolerance.

The first panel of Table 5 shows the optimal allocations for different levels of risk tolerance for the posited new expected returns and the new risk structure. The second panel shows the percentage holdings that each clientele will have once they establish their optimal allocations. Investors of clientele (a), who as explained above now control 6.24% of the total wealth, wish to allocate 76.74% of that wealth to cash. This means that after they make the trades necessary to establish this allocation they will hold 4.79% of the total market wealth in the form of cash (6.24% multiplied by 76.74%). As shown in the second panel of Table 5, the total allocation to cash across all clienteles is 14.29%, which is precisely the market weight of cash in the total economy. Thus supply of cash equals demand for cash. This is true of the other asset classes as well, which shows that the posited equilibrium expected returns in the second panel of Table 4 do indeed equate supply with demand.

At the opposite end of the spectrum to clientele (a), the most risk-tolerant clientele (g) wants to leverage up a further 15.86 percentage points in the crisis scenario to a level of 147% in order to take advantage of the opportunities presented by the high risk premiums on equities and bonds.

Table 5

	Clientele	(a)	(b)	(c)	(d)	(e)	(f)	(g)	
	Risk Tolerance	0.1	0.2	0.3	0.4	0.5	0.6	0.7	
	% of Total Wealth	6.24%	11.57%	21.32%	29.27%	18.69%	8.87%	4.05%	
New Optimal Allocations	US Equity	4.27%	8.54%	12.81%	17.08%	20.98%	25.18%	29.37%	
	Dev Equity	4.69%	9.39%	14.08%	18.77%	23.10%	27.72%	32.34%	
	Em Equity	3.75%	7.49%	11.24%	14.99%	19.45%	23.34%	27.23%	
	Bonds	10.55%	21.11%	31.66%	42.21%	41.52%	49.82%	58.13%	
	Cash	76.74%	53.48%	30.21%	6.95%	-5.05%	-26.07%	-47.08%	
	Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	
									Total
New % Holdings in Economy	US Equity	0.27%	0.99%	2.73%	5.00%	3.92%	2.23%	1.19%	16.33%
	Dev Equity	0.29%	1.09%	3.00%	5.49%	4.32%	2.46%	1.31%	17.96%
	Em Equity	0.23%	0.87%	2.40%	4.39%	3.64%	2.07%	1.10%	14.69%
	Bonds	0.66%	2.44%	6.75%	12.35%	7.76%	4.42%	2.35%	36.73%
	Cash	4.79%	6.19%	6.44%	2.03%	-0.94%	-2.31%	-1.91%	14.29%
	Total	6.24%	11.57%	21.32%	29.27%	18.69%	8.87%	4.05%	100.00%
Change in Allocations	US Equity	-1.20%	-0.31%	0.01%	-0.40%	0.66%	1.36%	-0.45%	
	Dev Equity	-1.24%	-0.22%	0.19%	-0.20%	0.53%	0.85%	-1.31%	
	Em Equity	-0.86%	0.05%	0.47%	0.27%	0.29%	-0.76%	-2.95%	
	Bonds	-4.57%	-3.35%	-3.73%	-6.13%	3.58%	19.83%	20.57%	
	Cash	7.87%	3.83%	3.05%	6.46%	-5.05%	-21.27%	-15.86%	
	Total	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	
									Total
Change in % Holdings in Economy	US Equity	-0.07%	-0.04%	0.00%	-0.12%	0.12%	0.12%	-0.02%	0.00%
	Dev Equity	-0.08%	-0.02%	0.04%	-0.06%	0.10%	0.08%	-0.05%	0.00%
	Em Equity	-0.05%	0.01%	0.10%	0.08%	0.05%	-0.07%	-0.12%	0.00%
	Bonds	-0.28%	-0.39%	-0.79%	-1.79%	0.67%	1.76%	0.83%	0.00%
	Cash	0.49%	0.44%	0.65%	1.89%	-0.94%	-1.89%	-0.64%	0.00%
	Total	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

One key observation that can be drawn from Table 5 is that despite the pronounced changes in market conditions and risk premiums, the amount of trading done in response is rather limited. Investors in all but the two most risk tolerant clienteles have a tactical response that involves less than 8% turnover. The total amount of turnover across all investors is only 7.44%.[8]

One puzzling result in Table 5 is that the risk tolerant investors substantially increase their holdings of bonds and actually decrease slightly their holdings of equities. This would seem to go against the notion that they are motivated to take more risk. Table 6 shows why this is happening. It shows the optimal portfolio of *risky* assets, i.e., equities and bonds, for each

[8]This is measured by summing up the absolute values of all the entries in the bottom panel of Table 5 and then dividing by 2. By dividing by 2 we do not double count by registering both a purchase and a sale.

clientele to hold. As shown in the top panel of Table 6, before the crisis all clienteles with risk tolerance less than or equal to 0.5 (clienteles (a) through (d)) hold the *risky* assets in the same proportions. Their asset allocations differ only in the amount allocated to the risky asset portfolio and the amount allocated to cash. The more risk tolerant clienteles hold the risky assets in slightly different proportions. This difference arises because we have assumed that these investors, who want to obtain higher returns through leverage, must borrow at a 50 basis point spread over the return on cash.

Table 6

	Clientele	(a)	(b)	(c)	(d)	(e)	(f)	(g)
	Risk Tolerance	0.2	0.3	0.4	0.5	0.6	0.7	0.8
	% of Total Wealth	5.00%	10.00%	20.00%	30.00%	20.00%	10.00%	5.00%
Optimal Risky Asset Portfolio Before Crisis	US Equity	20.96%	20.96%	20.96%	20.96%	23.26%	25.13%	25.13%
	Dev Equity	22.75%	22.75%	22.75%	22.75%	25.84%	28.35%	28.35%
	Em Equity	17.64%	17.64%	17.64%	17.64%	21.94%	25.43%	25.43%
	Bonds	38.64%	38.64%	38.64%	38.64%	28.96%	21.09%	21.09%
Optimal Leverage Before (Risky/Total)		39.87%	59.80%	79.73%	99.66%	100.00%	102.97%	117.69%
	Clientele	(a)	(b)	(c)	(d)	(e)	(f)	(g)
	Risk Tolerance	0.1	0.2	0.3	0.4	0.5	0.6	0.7
	% of Total Wealth	6.24%	11.57%	21.32%	29.27%	18.69%	8.87%	4.05%
Risky Asset Portfolio After Crisis but Before Adjustments	US Equity	17.57%	17.57%	17.57%	17.57%	20.32%	22.73%	22.73%
	Dev Equity	19.07%	19.07%	19.07%	19.07%	22.57%	25.64%	25.64%
	Em Equity	14.79%	14.79%	14.79%	14.79%	19.17%	23.00%	23.00%
	Bonds	48.58%	48.58%	48.58%	48.58%	37.94%	28.62%	28.62%
Leverage after Crisis (Risky/Total)		31.13%	50.35%	72.84%	99.51%	100.00%	104.79%	131.22%
New Optimal Risky Asset Portfolio	US Equity	18.35%	18.35%	18.35%	18.35%	19.97%	19.97%	19.97%
	Dev Equity	20.17%	20.17%	20.17%	20.17%	21.99%	21.99%	21.99%
	Em Equity	16.11%	16.11%	16.11%	16.11%	18.52%	18.52%	18.52%
	Bonds	45.37%	45.37%	45.37%	45.37%	39.52%	39.52%	39.52%
Optimal Leverage After (Risky/Total)		23.26%	46.52%	69.79%	93.05%	105.05%	126.07%	147.08%

Now consider the change in the composition of the optimal allocation to risky assets that is brought about by the crisis for the most risk tolerant investors. After equities fall by 40% and bonds fall by 10% in value, equities decline and bonds increase as a proportion of the optimal risky asset portfolio. This is shown in the middle panel of Table 6. Note that for the most risk tolerant, the fall in the value of risky assets has increased their effective leverage from

approximately 118% to 131%. Now consider the trading that the most risk tolerant will do to move to their optimal allocations, which are given in the bottom panel of Table 6. First, as can be seen the optimal portfolio of risky assets is more heavily weighted toward bonds (39.52%) than the before-adjustment portfolio of risky assets (28.62% is the weight before adjustment on bonds). Second, the most risk tolerant wish to further increase leverage from 131% to 147%. This means that the most risk tolerant will be borrowing more and using the proceeds to buy more of the optimal risky asset portfolio, which as explained above is skewed more heavily toward bonds than the current holdings. The consequence of all this is that the risk tolerant investors will buy relatively more bonds than equities.

3. Asset Allocation in the Financial Crisis: Alternative Crisis Scenarios

One might worry that the results reported in the last section could be quite sensitive to the assumptions we have made. In this section we consider some significant variations in our assumptions to see how robust our qualitative findings are to different scenarios. What we find is that the amount of turnover is generally quite limited across a broad range of assumptions that we can make. Tactical responses for almost all investors do not involve extreme moves. In fact, for most investors these changes are in fact quite modest. We also find that risk premiums are not very sensitive to most of the variations we consider, but not to all.

In our first variation we assume that investors cannot short-sell assets and in particular cannot borrow to achieve leverage. Intuitively this will limit the ability of the most risk tolerant investors to take on more risk in the crisis relative to what they could do in our base case scenario in which leverage was possible, as shown in Table 6. Table 7 shows the optimal allocations before and after the crisis when leverage is assumed to be infeasible or prohibitively expensive.

Table 7

	Clientele	(a)	(b)	(c)	(d)	(e)	(f)	(g)
	Risk Tolerance	0.2	0.3	0.4	0.5	0.6	0.7	0.8
	% of Total Wealth	5.00%	10.00%	20.00%	30.00%	20.00%	10.00%	5.00%
Optimal Allocations	US Equity	8.52%	12.78%	17.04%	20.92%	23.26%	25.60%	27.94%
	Dev Equity	9.20%	13.79%	18.39%	22.70%	25.84%	28.98%	32.13%
	Em Equity	6.96%	10.44%	13.91%	17.57%	21.94%	26.31%	30.68%
	Bonds	16.99%	25.48%	33.98%	38.81%	28.96%	19.10%	9.25%
	Cash	58.34%	37.50%	16.67%	0.00%	0.00%	0.00%	0.00%
	Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	Clientele	(a)	(b)	(c)	(d)	(e)	(f)	(g)
	Risk Tolerance	0.1	0.2	0.3	0.4	0.5	0.6	0.7
	% of Total Wealth	6.21%	11.50%	21.14%	29.24%	18.69%	8.94%	4.27%
New Optimal Allocations	US Equity	4.36%	8.73%	13.09%	17.26%	20.77%	24.28%	27.79%
	Dev Equity	4.78%	9.57%	14.35%	18.94%	22.87%	26.80%	30.73%
	Em Equity	3.52%	7.05%	10.57%	14.48%	19.69%	24.91%	30.12%
	Bonds	13.85%	27.70%	41.55%	49.31%	36.66%	24.01%	11.36%
	Cash	73.48%	46.95%	20.43%	0.00%	0.00%	0.00%	0.00%
	Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Change in Allocations	US Equity	-1.23%	-0.35%	-0.07%	-0.26%	0.45%	0.91%	1.08%
	Dev Equity	-1.26%	-0.22%	0.15%	-0.07%	0.30%	0.35%	0.03%
	Em Equity	-1.05%	-0.36%	-0.17%	-0.23%	0.53%	0.89%	0.79%
	Bonds	-2.89%	0.56%	2.19%	0.56%	-1.28%	-2.14%	-1.89%
	Cash	6.42%	0.36%	-2.10%	0.00%	0.00%	0.00%	0.00%
	Total	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

As can be seen in Table 7, trading is now more limited and tactical responses are less pronounced. In fact, total aggregate turnover is now only 1.68% rather than 7.44% as it was in the base case. A comparison between the no-leverage case and the base case is shown in Table 8.

Table 8

		Base case	No Leverage (A)	Equal Wealth Clienteles (B)	No Decrease in Risk Tolerance (C)	High Correlations (D)	Naïve Rebalancing (E)
Before							
	Average Risk Tol	0.500	0.500	0.500	0.500	0.500	0.500
Equilibrium Expected Returns	US Equity	7.11%	7.20%	7.23%	7.11%	7.11%	7.11%
	Dev Equity	7.60%	7.68%	7.72%	7.60%	7.60%	7.60%
	Em Equity	9.84%	9.93%	9.96%	9.84%	9.84%	9.84%
	Bonds	4.71%	4.79%	4.82%	4.71%	4.71%	4.71%
	Cash	3.00%	3.00%	3.00%	3.00%	3.00%	3.00%
	Sharpe Ratio	0.274	0.280	0.282	0.274	0.274	0.274
After							
	Average Risk Tol	0.385	0.387	0.371	0.485	0.385	0.385
Equilibrium Expected Returns	US Equity	13.53%	14.21%	14.09%	10.96%	16.14%	13.02%
	Dev Equity	13.63%	14.30%	14.19%	11.04%	16.18%	13.11%
	Em Equity	17.37%	18.03%	18.08%	14.01%	21.22%	16.67%
	Bonds	6.40%	7.11%	6.68%	5.30%	8.09%	6.34%
	Cash	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
	Sharpe Ratio	0.481	0.513	0.503	0.382	0.533	0.464
	Total Turnover	7.44%	1.68%	8.62%	5.64%	5.79%	8.23%

The results for the base case are given in the first column and those for the no leverage case are given in column (A). Note that when leverage is not feasible, risk premiums are higher. They are slightly higher before the crisis (equilibrium expected returns are approximately 10 basis points higher without leverage) and this difference is greater after the crisis (the difference is closer to 70 basis points). Table 8 also gives the Sharpe ratios pre- and post-crisis. In the base case the Sharpe ratio increases from 0.274 to 0.481 because of the crisis. When leverage is not allowed, Sharpe ratios are higher than they are in the base case (both before and after the crisis), since without leverage the risk tolerant investors are unable to take on as much risk as they were in the base case. This means less risk tolerant investors must be compensated to bear more of the risk.

In our second variation, Case B, we assume more heterogeneity among investors in terms of their risk tolerance. Whereas in the base case we assumed most investors before the crisis had risk tolerance close to 0.5, we now assume a uniform wealth distribution across all clienteles, i.e., each clientele is assumed to have one seventh of the aggregate wealth before the crisis. As shown in the third column of Table 8, risky asset returns and Sharpe ratios are higher in this case than in base case. The higher demand for risky assets due to the increased number of risk tolerant investors doesn't quite offset the higher supply by the increased numbers of risk-averse investors.[9] In this variation the amount of turnover due to tactical responses to the crisis is only modestly greater than it was in our bases case (8.62% rather than 7.44%). In our third variation, Case C, we assume that individuals do not become less risk tolerant because of the crisis. In other words, we do not decrement each clientele's risk tolerance by 0.1 as we do in the base case. Note that, even in this case, average risk tolerance does decrease in the crisis, but only from 0.5 to 0.485 and purely because the more risk tolerant lose relatively more wealth going into the crisis than the less risk tolerant. As would be expected, risk premiums on risky assets do not increase as much in the crisis relative to what occurs in the base case and the Sharpe ratio increases by much less than it does in the base case. This suggests that a key issue in inferring how risk premiums changed in the crisis is the degree to which investors became more fearful and less tolerant of risk. We should also emphasize that in all these variations we are taking the 40% decline in equities and 10% decline in bonds that we observed in the crisis as given. We know that some of the decline in value is due to downward revisions in cash flow projections and some to increases in risk premiums. This means that in Case C, where less of the decline can be attributed to increases in risk premiums, more must be attributed to downward revisions in cash flow projections. Note that in Case C we see again that the tactical responses to the crisis are not significant with aggregate turnover being less than what it was in the base case.

In our next variation, Case D, we assume that risk was even greater in the crisis due to correlations between asset class returns being even higher. Specifically, we assume that in the crisis correlations between the equity asset classes increase to 0.90 and the correlation between bonds and each equity asset class increases to 0.75. These higher correlations limit gains to diversification even further and, as would be expected, risk premiums after the crisis are larger in this case than they were in the base case, as is the Sharpe ratio.

Finally Case E, in our last variation, we assume that 25% of investors in each clientele follow a strategy in which they rebalance their portfolios to maintain the "target weights" they had before the crisis. Obviously it is not possible for all investors to rebalance to their pre-crisis allocation weights, but a subset of investors can do so. Investors following a fixed weight

[9] Technically, the risk tolerance parameter is in the denominator of the expected return equation, so the result is due to Jensen's inequality.

rebalancing strategy wish to buy equities (which have fallen most in value) and sell bonds and cash which have fallen by less or, in the case of cash instruments, appreciated. Sharpe (2009) points out that those who follow such a target-rate policy are acting as *de facto* contrarians with respect to the asset classes. If for example the policy weight for equities is 60%, and equities drop in value relative to fixed income and other assets in the mix, the target-weight plans will be buying equities to get back to the 60% weight. There is no reason to believe that the optimal solution is a policy that always involves rebalancing to a set of fixed weights established at some arbitrary time. In this scenario the fact that 25% of the investor population is following a contrarian strategy means that risk premiums do not increase as much as they do in the base case. As can be seen in Table 8, risk premiums on equities are about 50 basis points less than they are in the base case because of the contrarian trading. The contrarians act "as if" they have become more risk tolerant and are willing to absorb some of the net supply of risky assets offered by the risk intolerant.

4. *Summary and Discussion*

We have analyzed the tactical shifts in asset allocation that various investor clienteles should make in response to a crisis, imposing the requirement that these shifts be consistent with market equilibrium, i.e. the "laws of supply and demand." The crisis scenario we examined is one that approximately matches what we experienced in 2007 – 2009. We looked at allocations at the level of broad asset classes, *viz.* U.S. equities, developed markets equities, emerging markets equities, bonds, and cash. In the crisis scenario we considered, equities were assumed to lose 40% of their value and bonds to lose 10%. We assumed that asset volatility and cross-asset-class correlations increase substantially in the crisis and investors become less risk tolerant.

When investors have suffered losses approaching 40% of their wealth, when investment uncertainty has increased substantially, and when the ability to reduce risk through diversification has diminished, the natural inclination for most investors is to flee to safety by selling off risky assets and replacing them with safe assets. Unfortunately, not everyone can "flee to quality." Any investor who wants to flee to safety must find another investor who is willing to take the opposite side of the transaction and "flee to more risk." Prices must fall to the point where the rewards for bearing risk are high enough for the demand for risky assets to equal the supply. In our base case depiction of the crisis, we find that equity and bond risk premiums roughly triple to bring this about. The appropriate tactical allocation response for an investor to these changes depends critically on that investor's risk tolerance relative to the risk tolerance of the average investor. Investors who are more tolerant than the average investor in the market generally increase their holdings of the risky assets, accommodating the demand of the less risk tolerant investors who wish to do the opposite. This rebalancing occurs

notwithstanding the fact that more risk tolerant investors experience a higher "hit" to their wealth.

One of our key observations is that the appropriate tactical responses for most investors in a crisis are actually quite small. In our base case we find that for most investors the appropriate adjustment involves less than a 7% turnover. This finding is very robust to various changes we make in our assumptions. Only investors who are extremely risk averse or risk tolerant will find it appropriate to make more significant changes in their allocations. Even for these investors the adjustment is around 20%, not 90%. In our calibrated examples, the only way to justify the tactical response of an investor who makes a dramatic change from, say, 90% equities to 90% cash, would be to assume a very dramatic change in risk tolerance. Such an investor would need to be very exceptional, since if a significant number of investors did have such a dramatic change, there would need to be a significant number on the other side who had equally dramatic *increases* in risk tolerance.

Given our observation that the appropriate tactical response for most investors is quite limited, it follows that aggregate turnover will also be limited. In the various scenarios we consider we find that aggregate turnover ranges between 1.5% and 8.5%, depending on the assumptions we make. This is the trading that can be attributed to investors adapting their allocations to the risk/return tradeoff that emerges in the market after the crisis. Of course, we recognize that investors' motives for trade go beyond adjusting for risk and return. For example, in the crisis liquidity was a major concern for many investors. Some of the observed trading in the market was no doubt due to a subset of investors who needed liquidity and were selling liquid assets to generate needed cash – indeed, the needed liquidity was likely in part due to the "break down" of liquidity in other markets in the face of asymmetric information and counter-party risk.[10] The general results that we obtained above for the allocation of risky assets in the economy in the face of increasing risk can be carried over to a consideration to how illiquid assets are allocated after a major shock. Just as risky assets became more risky in the crisis, there was a sense in which illiquid assets became more illiquid. And just as there was an overall reduction in risk tolerance as investors became poorer and more cognizant of risk, there was in all likelihood an overall reduction in tolerance for illiquidity. This meant that just as risk premiums rose during the crisis, illiquidity premiums most likely rose as well. And just as there is an inclination for investors to flee to safety when risk increases, there is an inclination for investors to flee to liquidity when illiquidity becomes an even greater concern. Of course, the same equilibrium principle applies to illiquidity as it does to risk: an investor who wants to flee to liquid assets can only do so if some other investor is motivated to "flee to illiquidity." This will come about only if the illiquidity premium on illiquid assets increases sufficiently to equate

[10] Moreover, when brokers and intermediaries who play a role in equating demand and supply begin to be constrained in an illiquid crisis scenario, the evidence is that they "step back," and the ripple effects of that amplify the shifts in investor liquidity exposure across the clienteles.

the supply and demand of illiquid assets. Just as we had clienteles based on risk tolerance, there will be clienteles based on tolerance for illiquidity. Whether an investor is buying or selling illiquid assets depends on that investor's tolerance for illiquidity relative to the societal average.

Our analysis of asset allocation changes is "macroscopic" and thus closer to a starting point than an end point. It does not purport to capture all possible motivations for subsets of investors to adjust their portfolios, especially after a severe dislocation such as what we experienced in the last crisis. For example, most investors, both household and institutional, have non-traded assets (including human capital for household investors, opaque "alternative" assets for institutional funds like university endowments), and the risk and return characteristics of these no doubt changed in the crisis. Heterogeneity in exposures to these non-traded assets, and in the ability of institutional plan sponsors to meet their liability commitments, could provide some further motivations for changing asset allocations. Of course, these changes could not be unidirectional across all investors. Obviously one cannot argue that because of the increased risk of human capital, all investors should underweight equities. Once again the supply and demand principle must be observed: any investor who is making an allocation change because of human capital risk or non-traded asset risk must find another investor willing to make the opposite change. Trading will again be based on how the particular investor differs from the "average." With non-traded assets, of course, there may be no simple measure like risk tolerance to capture how one stands relative to the average, since human capital and other characteristics can vary along a number of dimensions. Nevertheless, although the situation is more complicated, the basic idea that an investor is not trading in a vacuum, and thus should consider how prices and returns are affected by the preferences of all investors, remains the same. Another motivation for adjustments in asset allocations in crisis situations may be related to taxes. Adjustments to changes in valuations and risk exposures may differ depending on whether an account is taxable or not, and this may create trading between the taxed and the non-taxed.

It is quite likely that a major explanation for much of the trading observed in the crisis was differences in investor beliefs, where investors agree to disagree and trade.[11] Two investors having the same levels of risk tolerance, same tolerances for illiquidity, same untraded asset positions, and same tax positions, may still find a reason to trade if they have different beliefs about future returns; in a crisis these different beliefs are more likely to be about macro or systemic factors that affect the performance of entire asset classes rather than about company specific fundamentals. The differing beliefs could even be due to (loosely) animal spirits or the behavioral biases of individuals as the never-experienced-before crisis situation unfolds.

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[11]We distinguish the "agree to disagree" differences in beliefs from situations in which investors are asymmetrically informed and less informed investors face potential losses due to trading with the better informed. Asymmetric information can result in less liquidity and less trading in markets and even lead to a complete market breakdown. This is in contrast to differing beliefs, which tend to increase trading.

Trading motivated by differing beliefs is in fundamental ways quite different from the trading we considered above. A trade in which a less risk tolerant investor reduces risk by trading with a more risk tolerant investor can be objectively viewed as a win/win transaction: both parties gain because the transaction results in better sharing of risk. The same case cannot be made as easily for transactions based on differing beliefs. Behind such a transaction is the belief by each party that the other party is on the wrong side of the trade. Objectively both parties cannot be right. Despite this it appears that a fair amount of trade is motivated by the belief that the market consensus is wrong. One can trade on this belief, but in doing so one should remember that even here the laws of supply and demand hold: essentially anyone trading on the basis of the market being "wrong" must find some other investor who also wants to trade on the market being "wrong," just "wrong" in the opposite direction. In the same vein, investors can't on average rely on the belief that they can "trade faster" than the average investor in adjusting their allocations to cut their losses as a form of risk control in a crisis scenario.[12]

We have intentionally kept the examples used in our analysis as simple as possible to illustrate the fundamental and simple observations we wish to make. The main point to be made is that any tactical portfolio adjustments that investors wish to make in response to changed market conditions take place in a market where the laws of supply and demand govern. Prices and risk premiums adjust so that as a rough approximation one can say that the "average investor" will not want to trade. The trades that any particular investor will want to make depend on how that investor's risk preferences and other characteristics compare to the average investor. As we showed, for most investors tactical adjustments will be rather modest, even for large changes in market conditions. This is because much of the adjustment to new conditions comes through changes in prices and risk premiums.

[12] Indeed, it is estimated that, in recent "normal times", some 70% of trade volume in the U.S. is due to high-frequency-trading (HFT) which has more to do with uneven advances in trading technology than rebalancing demands. There is also some evidence that the HFT pulls back in fast-moving "crisis" markets. But we don't have a market microstructure level of granularity in our analysis, which is why we shouldn't try to compare turnover just due to rebalancing here with trade volume due to all possible causes.

REFERENCES

1) Bateman, Hazel, Jordan Louviere, Stephen Satchell, Towhidul Islam, and Susan Thorp, 2010, "Retirement investor risk tolerance in tranquil and crisis periods: experimental survey evidence," School of Finance and Economics, University of Technology, Sydney, March.

2) Marsh, Terry, and Paul Pfleiderer, 2011, Financial Modeling and the Crisis I: "Black Swans," Manuscript, Stanford University.

3) Sharpe, William F., 1974, "Imputing Expected Security Returns from Portfolio Composition," *Journal of Financial and Quantitative Analysis*, 9(4), 463-472.

4) Sharpe, William F., 2007, *Investors and Markets: Portfolio Choices, Asset Prices, and Investment Advice (Princeton Lectures in Finance)*, Princeton University Press, Princeton NJ.

5) Sharpe, William F., 2009, "Adaptive Asset Allocation Policies," *Financial Analysts Journal*, May/June, 66(3), 45-59.